Dynamic Posted Price Mechanisms

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Abstract

We consider the problem of selling a single commodity in unlimited supply, e.g., a digital good, by a dynamic "posted price" mechanism. In such a mechanism, each consumer is offered a price for their receipt of an item and they can choose to accept or reject the offer. The mechanism can use previous consumers' responses when setting prices for subsequent consumers. We show that a two round posted price mechanism can come within a factor of four of optimal for mass markets. Thus the penalty for using a dynamic posted price instead of running an auction is also at most a factor of four.

1 Introduction

Motivated by a number of Internet related problems, there has been a significant amount of research on optimization algorithms designed to run on data sets supplied by a collection of selfish autonomous agents. These selfish agents may report incorrect data values to manipulate the solution of the algorithm. Standard *public value* algorithms fail to work when the data is the *private values* of selfish agents. A popular solution technique, borrowed from the field of mechanism design, is to look for *direct revelation* mechanisms that are *truthful* (a.k.a., incentive compatible or strategy-proof) where revealing their exact values truthfully is a dominant strategy for each agent [11, 1, 4, 6, 2, 7]. This is a compelling solution as the revelation principle states that if there is a dominant strategy mechanism, there is a truthful one [9].

As a concrete example and the problem discussed in the remainder of this paper, consider selling a single commodity available in unlimited supply, e.g., a digital good like pay-per-view TV. Here the agent's private value is their *utility*, the most they are willing to pay for the item. Truthful mechanisms as solutions to this problem have been studied for both the single round (offline) [9, 6, 13] and online [2] cases.

A drawback of truthful direct revelation mechanisms is that they require the agents to communicate their exact value to the mechanism [3]. This may be a problem for a number of reasons. An agent may not know their exact value and computing it may be expensive [8], or due to a lack of trust of the mechanism or to keep their private information private agents may not believe that playing truthfully is their best strategy [10]. For these reasons we consider posted price mechanisms,[1] mechanisms where each agent receives a posted price for the commodity and can choose to either accept or reject it. We will restrict our attention to mechanisms that only get one shot with each agent. We will allow the mechanism to base the price it offers some agents on the response given by other agents. It is easy to see that in a posted price mechanism each agents best strategy is to accept the price if it is below their utility value and reject the price if it is above their utility value.

As in the truthful auctions of [6] we will compare the performance of a posted price mechanism to the revenue of *optimal fixed pricing*, the selling mechanism that chooses the optimal price to sell the commodity given perfect market information. We give a two round posted price selling mechanism that asymptotically achieves a factor of four of optimal fixed pricing for mass markets, i.e., when the number of sold items by optimal fixed pricing is large. This contrasts with the auctions of [5] that are asymptotically optimal in this case. Thus,

[1] Posted price selling mechanisms are standard on the Internet and in conventional market places. Though, using posted prices as a mechanism for market research as we do in this paper is not as common and attempts to do so have been met with consumer resistance [12].

for mass markets, the penalty for using the weaker posted price mechanism instead of a direct revelation truthful mechanism is at most a factor of four.

2 Main Result

We assume that the seller has a bound on the range of utility values to be in $[1, h]$. A single round mechanism can achieve a $\log h$ approximation to optimal fixed pricing by picking $p_i = 2^K$ for K chosen uniformly at random from $\{1, \ldots, \log h\}$. It is not possible to do better than this with a single round mechanism.

We now give a two round mechanism that is constant competitive with the optimal fixed pricing on mass markets.

Round 1: Partition the agents into two sets uniformly at random, S and T. Further randomly partition S into $\log h$ sets, $S^{(1)}, \ldots, S^{(\log h)}$. For $i \in \{1, \ldots, \log h\}$ offer agents in $S^{(i)}$ price 2^i and tally their responses.

Round 2: Let i^* be the index of the set $S^{(i)}$ that achieves the most revenue in Round 1. Offer agents in T price 2^{i^*} and tally their responses.

THEOREM 2.1. *As the number of agents grows, the above mechanism converges to achieve an expected revenue of at least a fourth of the revenue of optimal fixed pricing.*

The proof of this is a simple exercise in using the Chernoff bound in a similar fashion to [5].

3 Conclusions

It is interesting that in the mass market sale of a single commodity in unlimited supply, posted price mechanisms can perform within a constant factor of the best truthful direct revelation mechanisms. It leaves a natural open question as to whether other mechanism design problems permit posted price mechanisms as a solution. In particular, it would be interesting to consider a posted price mechanism for the "online" auction problem of [2].

We can also consider mechanisms that are allowed to ask multiple questions to each agent about their values. For the single commodity seller, for example, an adaptation of the model that allows the seller make multiple offers to the same agent will retain its dominant strategy properties as long as the offer prices to an agent are increasing and that the seller stop making offers after the first rejection by that agent.

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